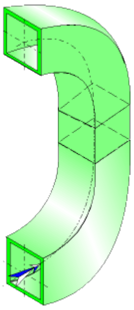
U-shaped Bends
(with flow in one plane)
Rectangular Cross-Section
(IDELCHIK)



Model description:

This model of component calculates the head loss (pressure drop) of U-shaped bends (with flow in one plane) whose cross-section is rectangular and constant. In addition, the flow is assumed fully developed and stabilized upstream of the first bend.

Model formulation:

Hydraulic diameter (m):

$$D_h = \frac{2 \cdot a_0 \cdot b_0}{a_0 + b_0}$$

([1] diagram 6-1)

Cross-section area (m^2):

$$F_0 = a_0 \cdot b_0$$

Total length measured along the axis (m):

$$l = 2 \cdot \left(2 \cdot \pi \cdot R_0 \cdot \frac{\delta}{360} \right) + l_{el}$$

Mean velocity (m/s):

$$w_0 = \frac{Q}{F_0}$$

Mass flow rate (kg/s):

$$G = Q \cdot \rho$$

Fluid volume (m^3):

$$V = F_0 \cdot l$$

Fluid mass (kg):

$$M = V \cdot \rho$$

Reynolds number:

$$\mathrm{Re} = \frac{w_0 \cdot D_h}{\nu}$$

Relative roughness:

$$\overline{\Delta} = \frac{\Delta}{D_h}$$

■ Case of relative radius of curvature lower than 3 ($R_0/b_0 < 3$) ([1] diagram 6-1)

Coefficient of effect of the roughness:

$$k_\Delta = f\left(\frac{R_0}{b_0}, \mathrm{Re}, \overline{\Delta}\right)$$

([1] diagram 6-1)

- $0.50 \le R_0/b_0 \le 0.55$

$\overline{\Delta}$	Re	
	$3 \cdot 10^3$ - $4 \cdot 10^4$	> $4 \cdot 10^4$
0	1.0	1.0
0 - 0.001	1.0	$1 + 0.5 \cdot 10^3 \cdot \overline{\Delta}$
> 0.001	1.0	1.5

- $R_0/b_0 > 0.55$

$\overline{\Delta}$		Re	
	$3 \cdot 10^3$ - $4 \cdot 10^4$	> $4 \cdot 10^4$ - $2 \cdot 10^5$	> $2 \cdot 10^5$
0	1.0	1.0	1.0
0 - 0.001	1.0	$\lambda_\Delta / \lambda_{sm}$	$1 + 10^3 \cdot \overline{\Delta}$
> 0.001	1.0	2.0	2.0

with:

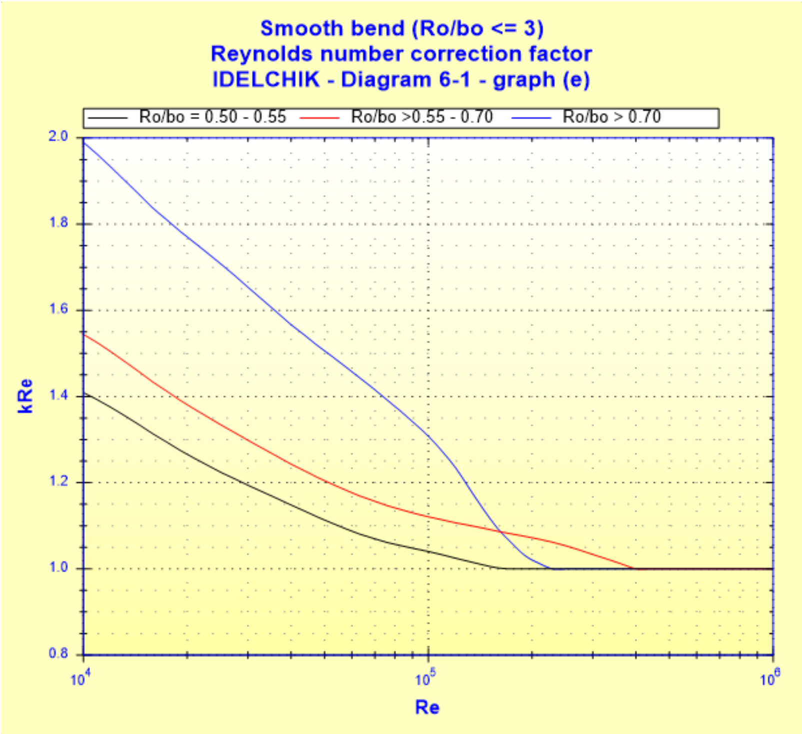
λ_{sm} : Darcy friction factor for hydraulically smooth pipe ($\overline{\Delta}$ = 0) at Re

λ_Δ : Darcy friction factor for rough pipe ($\overline{\Delta} = \Delta/D_h$) at Re

Coefficient of effect of the Reynolds number ($\mathrm{Re} \ge 10^4$):

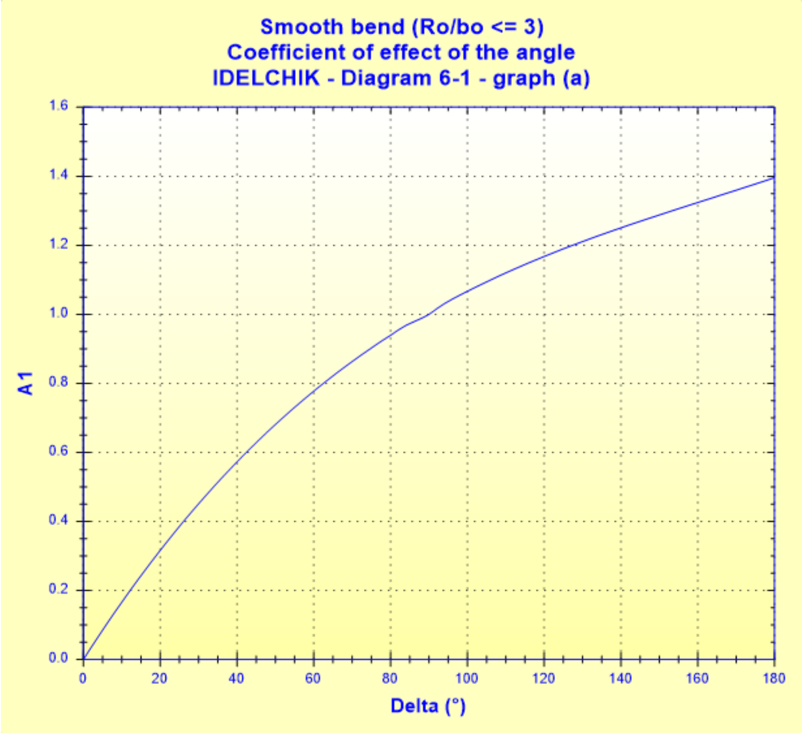
$$k_{\mathrm{Re}} = f\left(\mathrm{Re}, \frac{R_0}{b_0}\right)$$

([1] diagram 6-1)



Coefficient of effect of the angle:

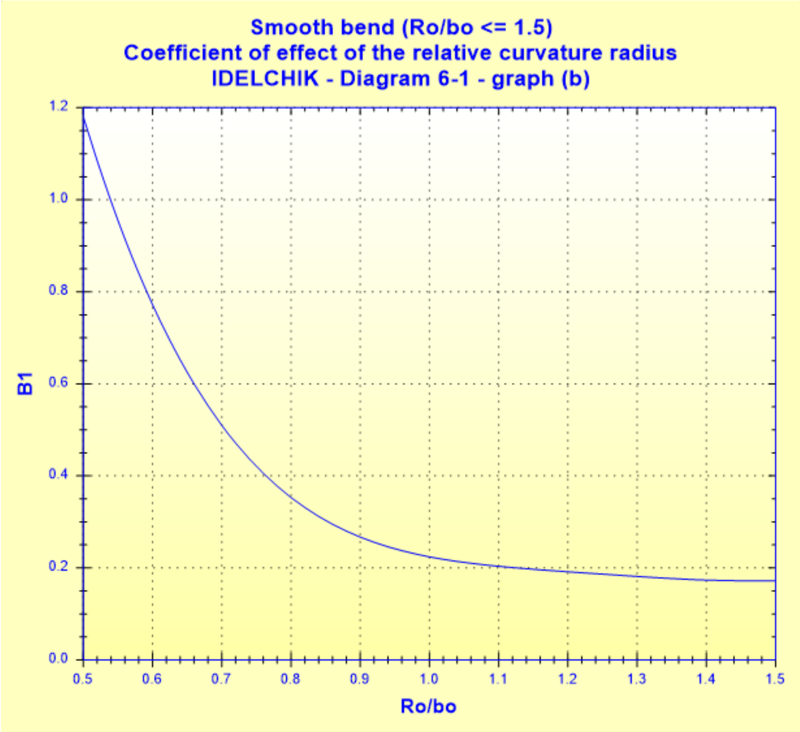
$$A1 = f(\delta)$$ ([1] diagram 6-1)



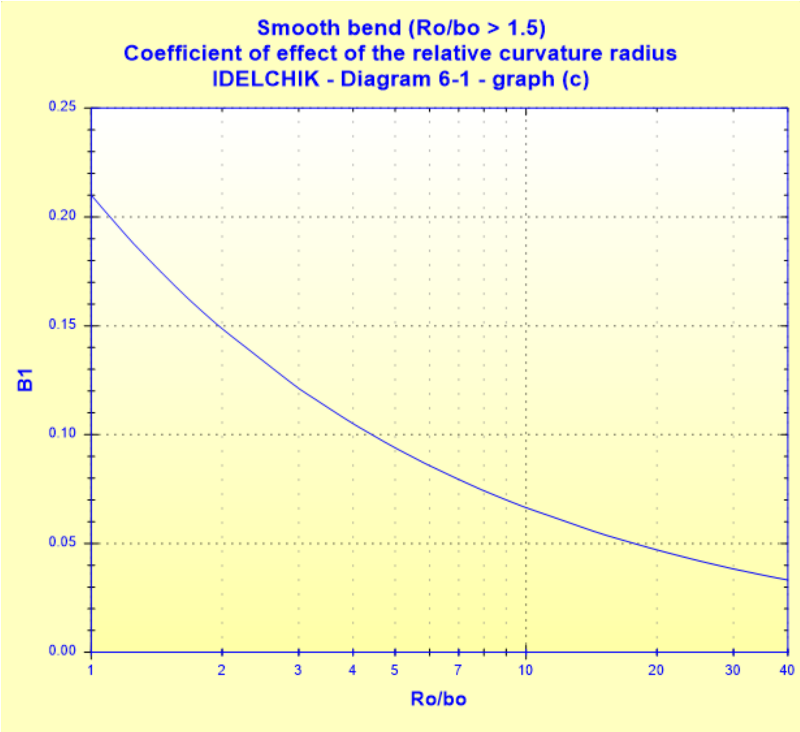
Coefficient of effect of the relative curvature radius:

$$B1 = f\left(\frac{R_0}{b_0}\right)$$ ([1] diagram 6-1)

- $0.5 \leq R_0/b_0 \leq 1.5$



- $R_0/b_0 > 1.5$



Coefficient of effect of the relative elongation of the cross section:

- $a_0 \geq b_0$

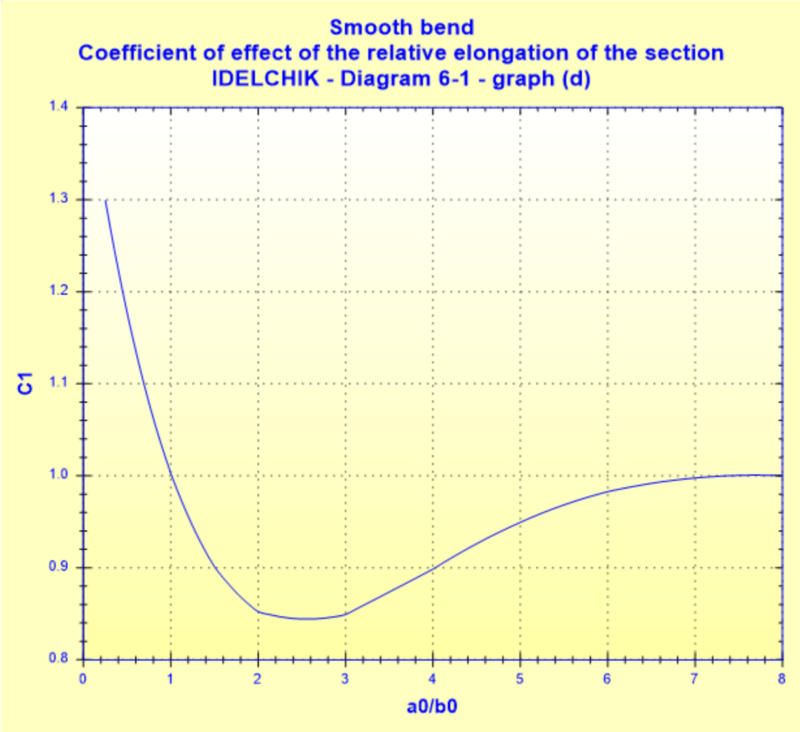
$$C1 = f\left(\frac{a_0}{b_0}\right)$$

([1] diagram 6-1)

- $a_0 < b_0$

$$C1 = f\left(\frac{b_0}{a_0}\right)$$

([1] diagram 6-1)

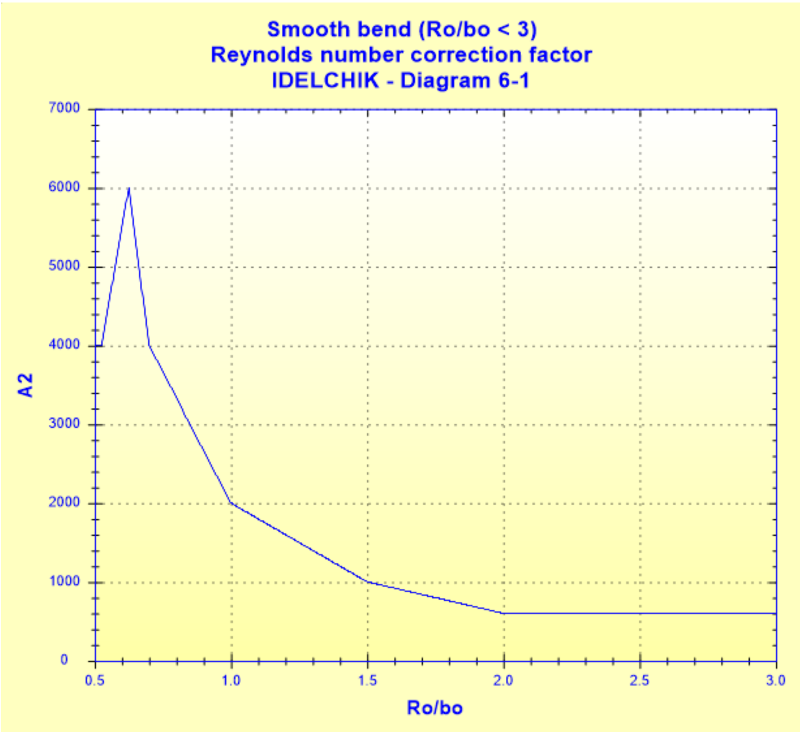


Reynolds number correction factor that depends on the relative curvature radius:

$$A2 = f\left(\frac{R_0}{b_0}\right)$$

([1] diagram 6-1)

R_0/b_0	0.50 - 0.55	>0.55 - 0.70	>0.70 - 1.0	>1.0 - 2.0	>2.0
$A2 \times 10^{-3}$	4.0	6.0	4.0 - 2.0	1.0	0.6



Pressure loss coefficient (without friction):

- $Re \geq 10^4$

$$\zeta'_{loc} = k_\Delta \cdot k_{Re} \cdot A1 \cdot B1 \cdot C1$$

([1] diagram 6-1)

● $3 \cdot 10^3 < Re < 10^4$

$$\zeta'_{loc} = \frac{A2}{Re} + A1 \cdot B1 \cdot C1$$

([1] diagram 6-1)

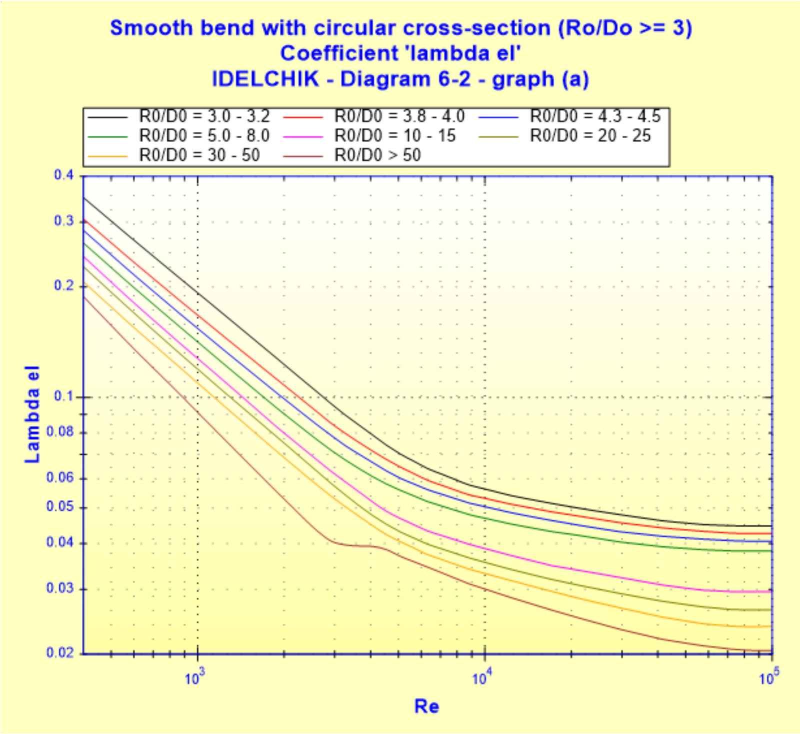
■ Case of relative radius of curvature greater than or equal to 3 ($R_0/b_0 \geq 3$) ([1] diagram 6-2)

Total friction factor with smooth wall:

● $4 \cdot 10^2 \leq Re < 10^5$

$$\lambda_{el} = f\left(Re, \frac{R_0}{D_0}\right)$$

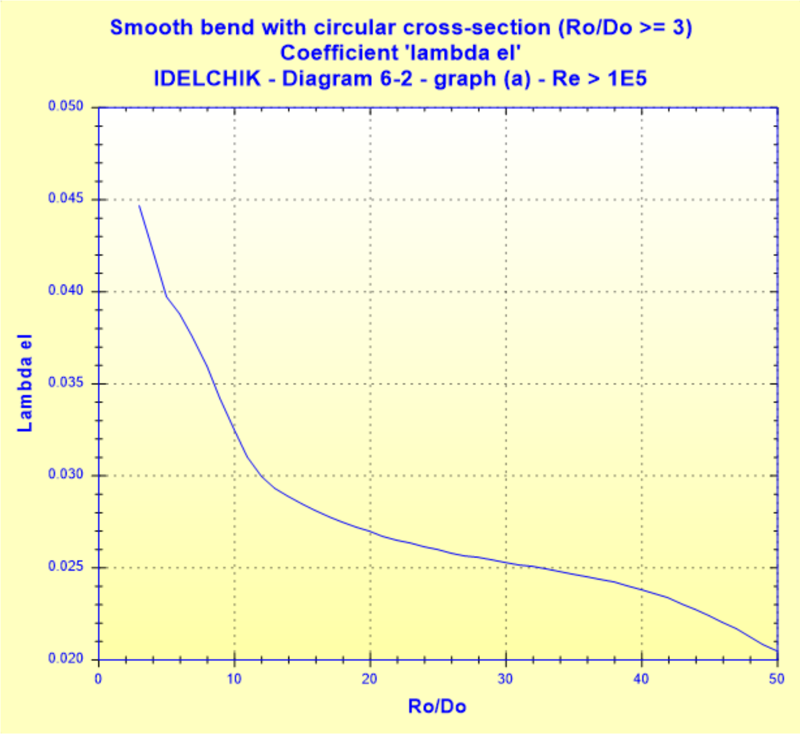
([1] diagram 6-2)



● $Re \geq 10^5$

$$\lambda_{el} = f\left(\frac{R_0}{D_0}\right)$$

([1] diagramme 6-2)



Estimation of the coefficient of local resistance

$$\zeta'_{loc} = (\lambda_{el} - \lambda_s) \cdot \frac{2 \cdot \pi \cdot R_0 \cdot \delta / 360}{D_h}$$

with:

λ_s : Darcy friction factor for hydraulically smooth pipe ($\bar{\Delta}$ = 0) at Re

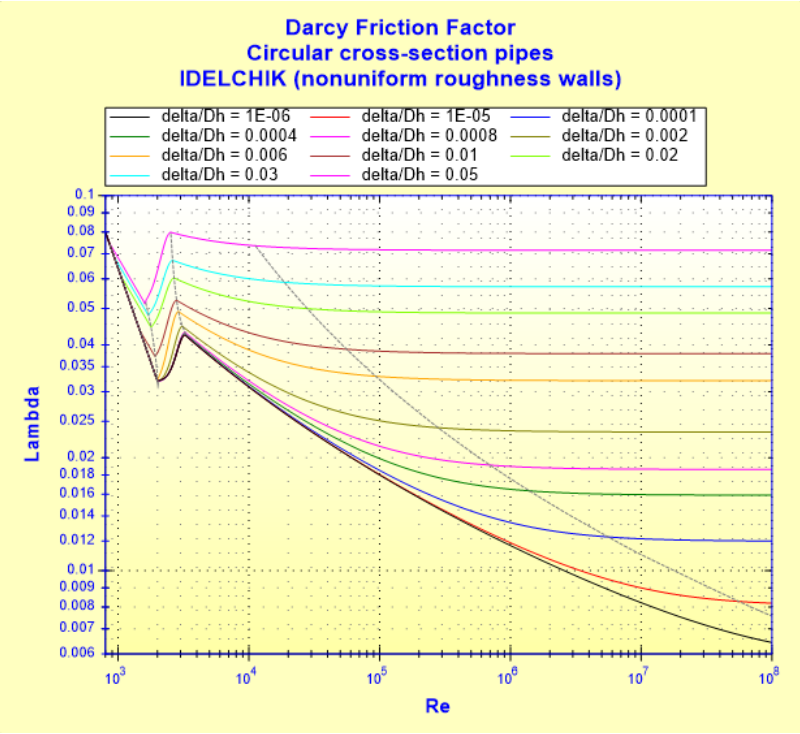
■ Case of the U-shaped Bends ([1] diagram 6-20)

Darcy friction factor:

See Straight Pipe - Rectangular Cross-Section and Nonuniform Roughness Walls (IDELCHIK)

■ Darcy friction factor for circular cross-section

$$\lambda_{circ} = f\left(\mathrm{Re}, \frac{\Delta}{D_h}\right)$$



■ Correction for Darcy friction factor for noncircular cross-section

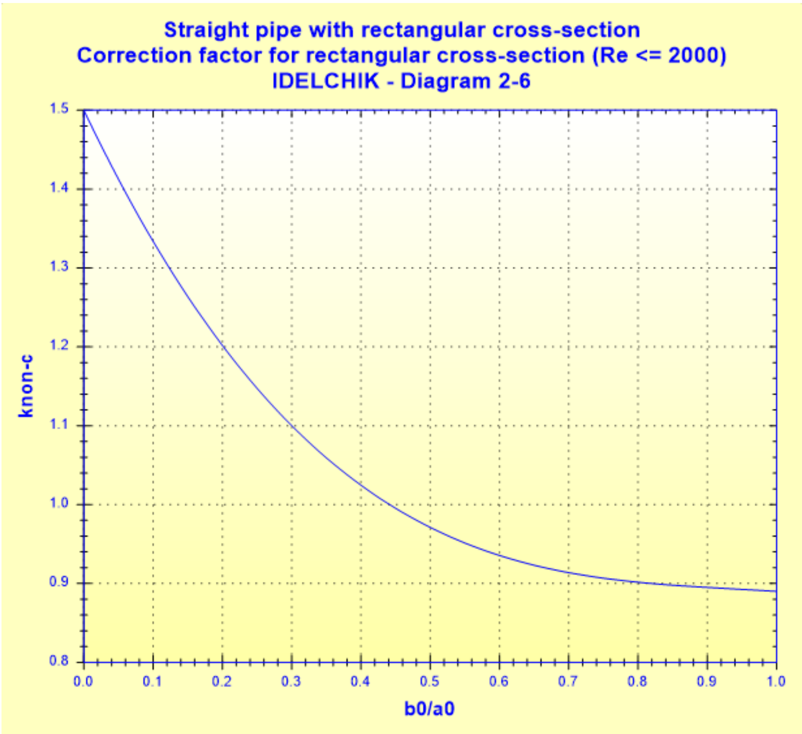
♦ $a_0 \geq b_0$

$$k_{non-c} = f\left(b_0/a_0\right)$$ ([1] diagram 2-6)

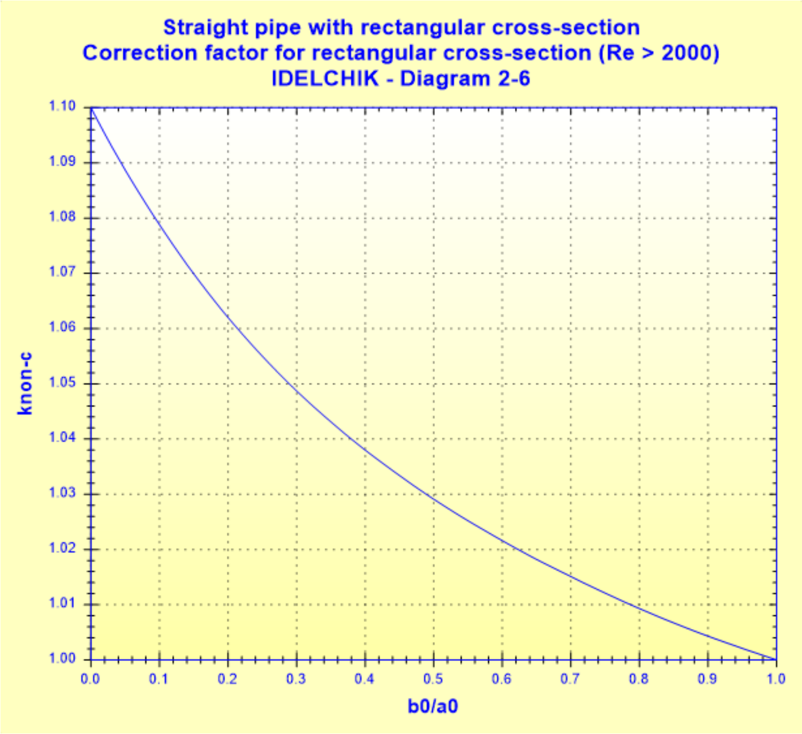
♦ $a_0 < b_0$

$$k_{non-c} = f\left(a_0/b_0\right)$$ ([1] diagram 2-6)

● laminar flow (Re ≤ 2000):



● turbulent flow (Re > 2000):



■ Darcy friction factor for rectangular cross-section

$$\lambda_{rect} = \lambda_{circ} \cdot k_{non-c}$$

([1] diagram 2-6)

Pressure loss friction factor:

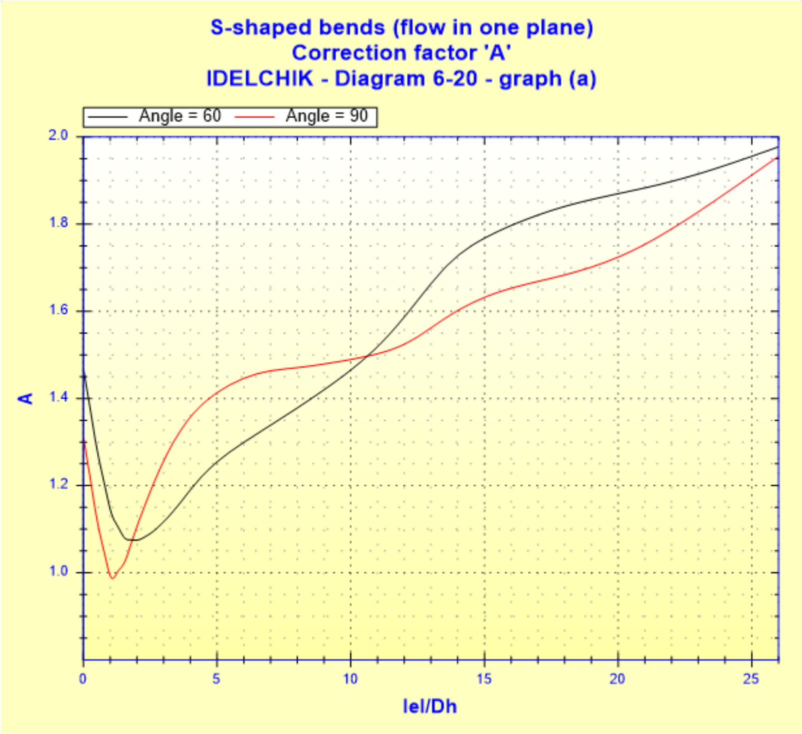
$$\zeta_{fr} = \lambda \cdot \left[2 \cdot \left(0.0175 \cdot \delta \cdot \frac{R_0}{D_h} \right) + \frac{l_{el}}{D_h} \right]$$

([1] diagram 6-20)

Interaction correction factor:

$$A = f\left(\frac{L_{el}}{D_h}, \delta \right)$$

([1] diagram 6-20 graph a)



Total pressure loss coefficient (based on the mean velocity in the bends):

$$\zeta = A \cdot \zeta'_{loc} + \zeta_{fr}$$

([1] diagram 6-20)

Total pressure loss (Pa):

$$\Delta P = \zeta \cdot \frac{\rho \cdot w_0^{\;2}}{2}$$

([1] diagram 6-20)

Total head loss of fluid (m):

$$\Delta H = \zeta \cdot \frac{w_0^{\;2}}{2 \cdot g}$$

Hydraulic power loss (W):

$$Wh = \Delta P \cdot Q$$

Straight length of equivalent pressure loss (m):

$$L_{eq} = \zeta \cdot \frac{D_h}{\lambda_{rect}}$$

Symbols, Definitions, SI Units:

a_0	Rectangular cross-section width (m)
b_0	Rectangular cross-section height (m)
D_h	Bend hydraulic diameter (m)
F_0	Cross-sectional area (m²)
l	Total length measured along the axis (m)
R_0	Radius of curvature (m)
δ	Curvature angle of each bend (°)
Q	Volume flow rate (m³/s)
w_0	Mean velocity (m/s)
G	Mass flow rate (kg/s)
V	Fluid volume (m³)
M	Fluid mass (kg)
Re	Reynolds number ()
Δ	Absolute roughness of walls (m)
$\overline{\Delta}$	Relative roughness of walls ()
k_Δ	Coefficient that allows for the effect of the roughness ()
k_{Re}	Coefficient that allows for the effect of the Reynolds number ()
A1	Coefficient that allows for the effect of the angle ()
B1	Coefficient that allows for the effect of the relative curvature radius ()
C1	Coefficient that allows for the effect of the relative elongation of the cross section ()
A2	Reynolds number correction factor that depends on the relative curvature radius ()
ζ'_{loc}	Coefficient of local resistance ()
λ_{circ}	Darcy friction coefficient for circular cross-section ()

k_{non-c}	Correction for Darcy friction factor for noncircular cross-section ()
λ_{rect}	Darcy friction coefficient for rectangular cross-section ()
λ_{el}	Friction coefficient ()
ζ_{fr}	Pressure loss friction factor ()
A	Interaction correction factor ()
ζ	Total pressure loss coefficient (based on the mean velocity in the bend) ()
ΔP	Total pressure loss (Pa)
ΔH	Total head loss of fluid (m)
Wh	Hydraulic power loss (W)
L_{eq}	Straight length of equivalent pressure loss (m)
ρ	Fluid density (kg/m³)
ν	Fluid kinematic viscosity (m²/s)
g	Gravitational acceleration (m/s²)

Validity range:

- stabilized flow upstream bend
- length of the straight section downstream: $\geq 10\ D_h$
- relative radius of curvature greater than or equal to 1 ($R_0/b_0 \geq 1$)
- curvature angle of one bend: 0 to 180°

  for 'δ' angles less than 60° the pressure loss coefficient 'ζ' is estimated by taking into account an interaction correction factor 'A' corresponding to that of an angle of 60°.

  for 'δ' angles greater than 90° the pressure loss coefficient 'ζ' is estimated by taking into account an interaction correction factor 'A' corresponding to that of an angle of 90°.

■ case of relative radius of curvature lower than 3 ($R_0/b_0 < 3$)

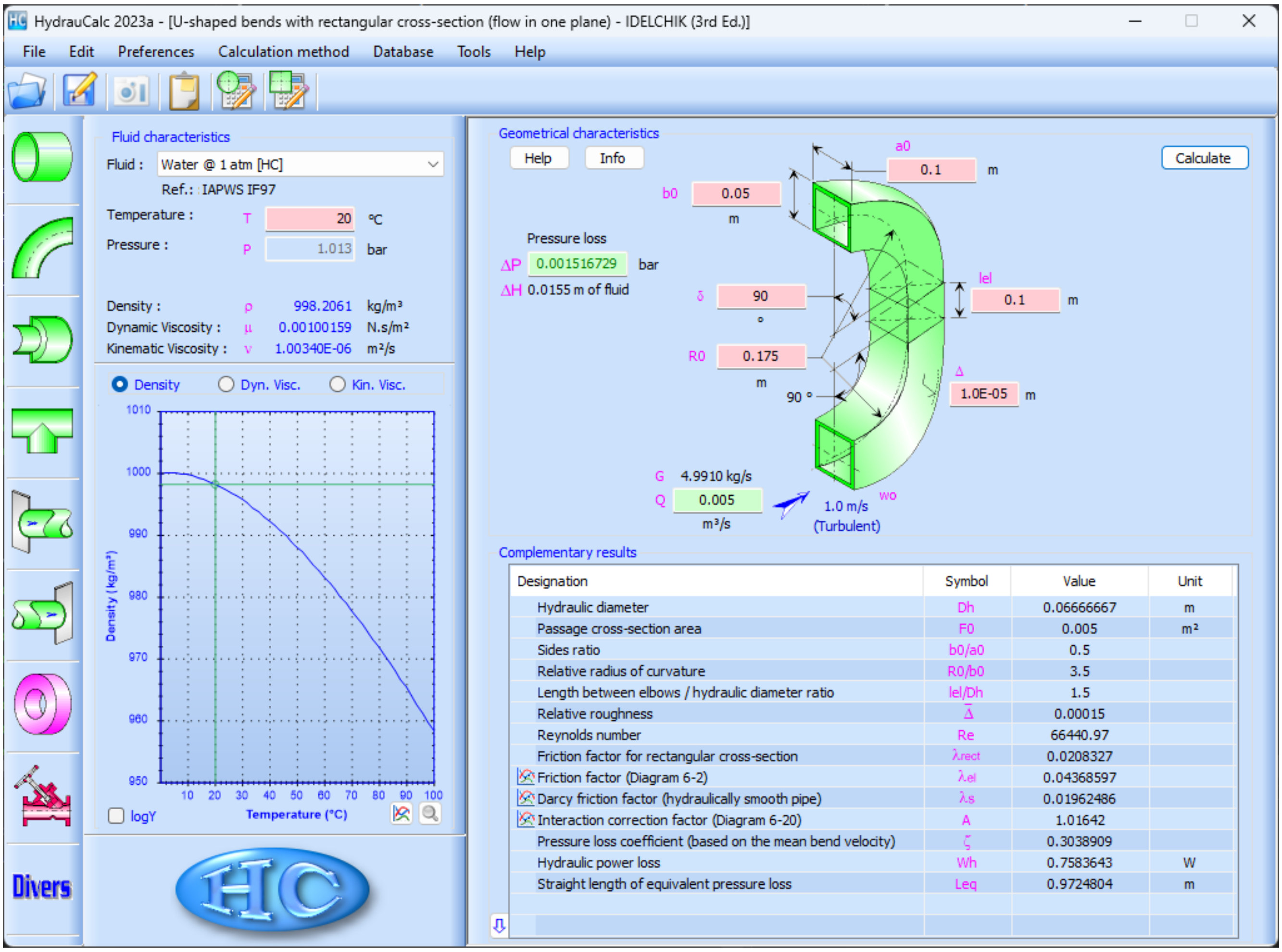
- flow regime: $Re \geq 3 \cdot 10^3$

■ case of relative radius of curvature greater than or equal to 3 ($R_0/b_0 \geq 3$)

- flow regime: $500 \leq Re \leq 38 \cdot 10^3$

  for Reynolds number 'Re' lower than 500 or greater than $38 \cdot 10^3$, the coefficient 'λ_{el}' is linearly extrapolated.

Example of application:



References:

[1] Handbook of Hydraulic Resistance, 3rd Edition, I.E. Idelchik