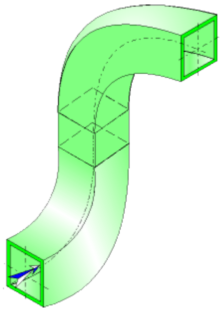
S-shaped Bends
(with flow in two perpendicular planes)
Rectangular Cross-Section
(MILLER)



Model description:

This model of component calculates the head loss (pressure drop) of S-shaped bends (with flow in two perpendicular planes) whose cross-section is rectangular and constant. In addition, the flow is assumed fully developed and stabilized upstream of the first bend.

An option allows to take into account the effect of the straight length at the exit of the second bend.

Model formulation:

Hydraulic diameter (m):

$$\boxed{D = \frac{2 \cdot b \cdot W}{b + W}}$$ ([1] equation 9.5)

Cross-section area (m^2):

$$\boxed{A = b \cdot W}$$

Mean velocity (m/s):

$$\boxed{U = \frac{Q}{A}}$$

Total length measured along the axis (m):

$$\boxed{L = 2 \cdot \left(2 \cdot \pi \cdot r \cdot \frac{\theta_b}{360}\right) + L_s}$$

Mass flow rate (kg/s):

$$\boxed{G = Q \cdot \rho}$$

Fluid volume (m^3):

$$\boxed{V = A \cdot L}$$

Fluid mass (kg):

$$\boxed{M = V \cdot \rho}$$

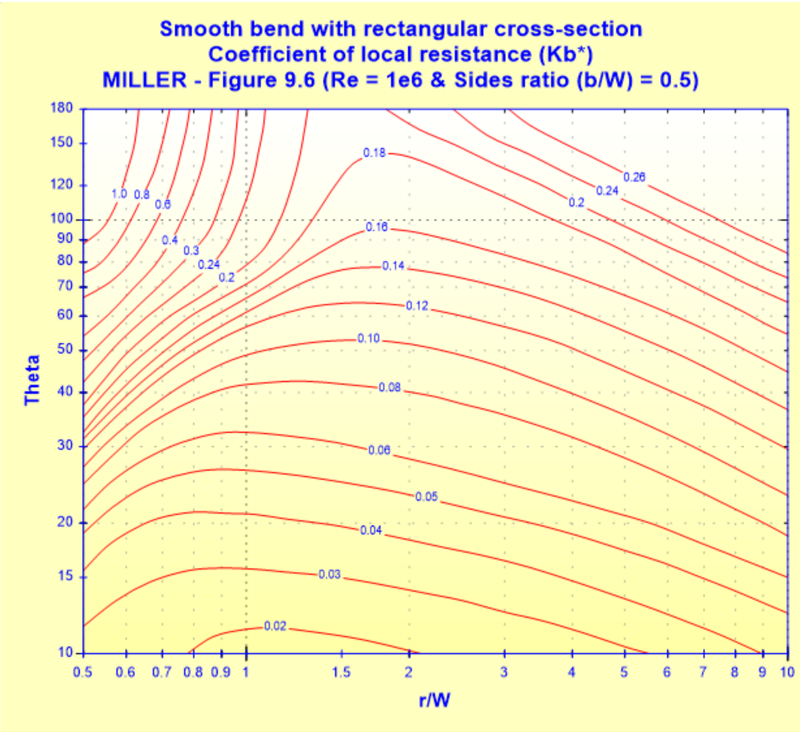
Reynolds number:

$$\boxed{Re = \frac{U \cdot D}{\nu}}$$ ([1] equation 9.6)

Basic resistance coefficient:

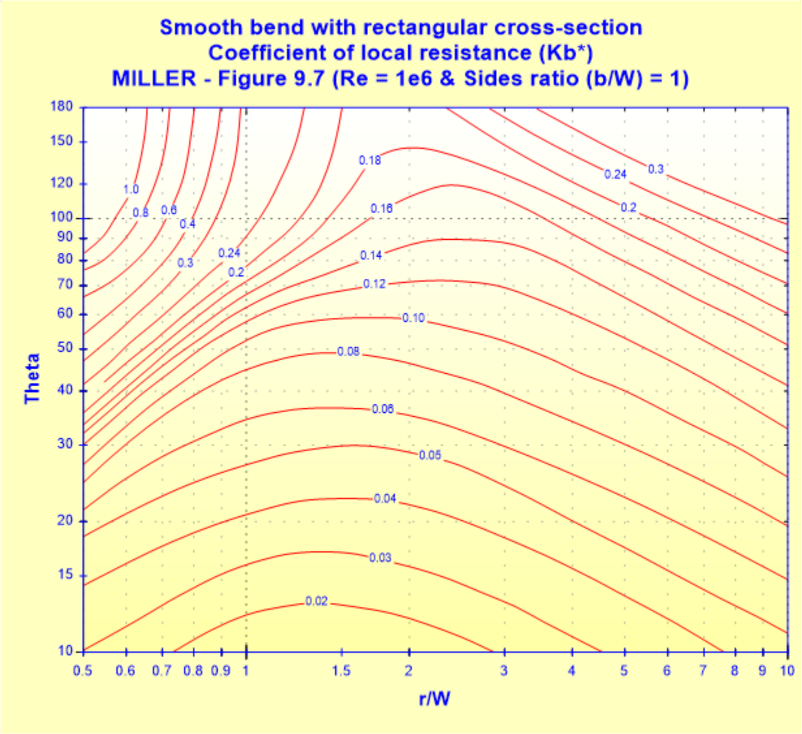
$$\boxed{K_b^* = f\left(\frac{r}{W}, \theta_b\right)}$$ ([1] figures 9.6 – 9.7 – 9.8)

■ Sides ratio b/W = 0.5



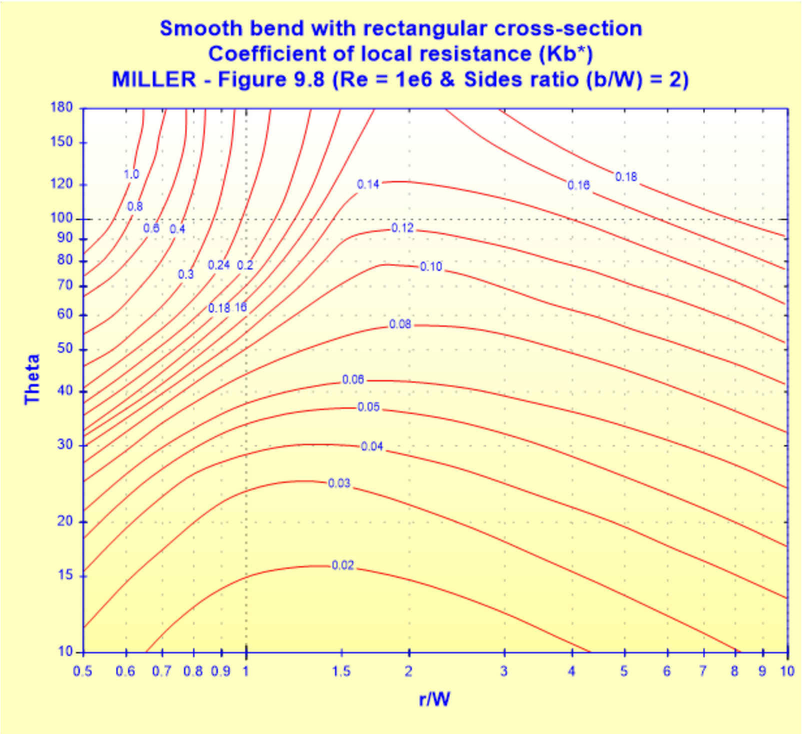
([1] figure 9.6)

■ Sides ratio b/W = 1



([1] figure 9.7)

■ Sides ratio b/W = 2



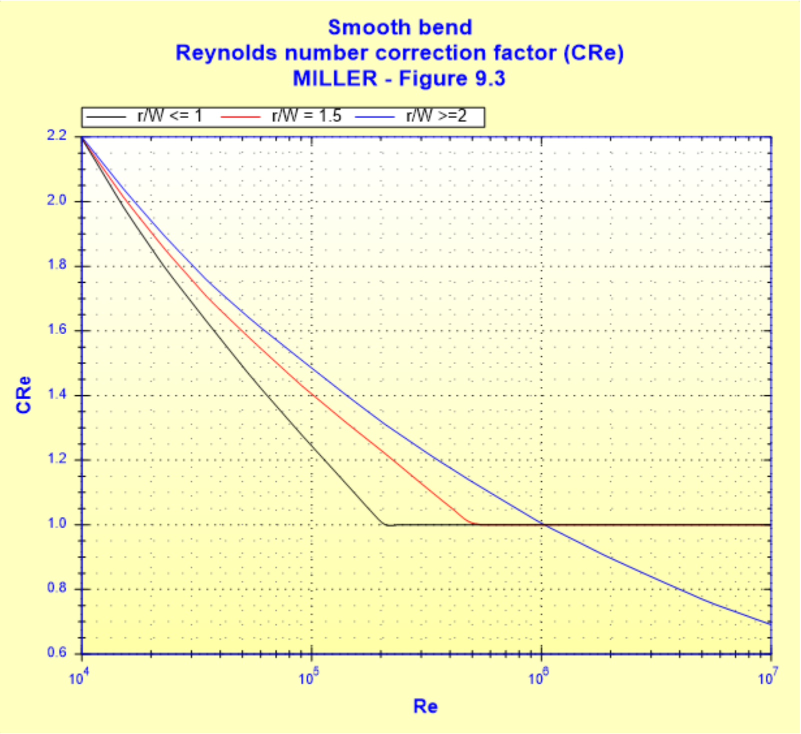
([1] figure 9.8)

For any sides ratio 'b/W' between 0.5 and 2, the coefficient K_b^* is obtained by curvilinear interpolation between the values of K_b^* calculated for aspect ratios of 0.5, 1 and 2.

Reynolds number correction factor:

$$\boxed{C_{Re} = f\left(\mathrm{Re}, \frac{r}{W}\right)}$$

([1] figure 9.3)



- $r/W \geq 1$

$$\boxed{C_{Re} = f\left(\text{Re}, \frac{r}{D}\right)}$$ ([1] figure 9.3)

- $r/W < 1$
  - $r/W > 0.7$ or $K_b^* < 0.4$

$$\boxed{C_{Re} = f\left(\text{Re}, \frac{r}{D}\right)}$$ ([1] figure 9.3 with r/W=1)

  - otherwise ($r/W \leq 0.7$ and $K_b^* \geq 0.4$)

$$\boxed{C_{Re} = \frac{K_b^*}{K_b^* - 0.2C'_{Re} + 0.2}}$$ ([1] equation 9.2)

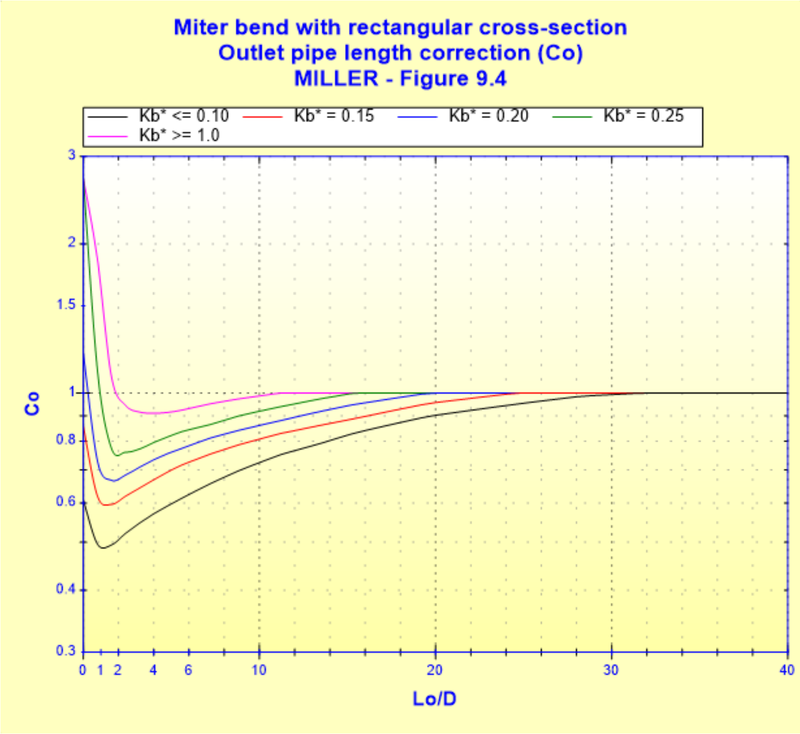
with:

$$\boxed{C'_{Re} = f\left(\text{Re}, \frac{r}{D}\right)}$$ ([1] figure 9.3 with r/W=1)

Outlet pipe length correction factor (optional):

- correction factor for circular cross-section
  - $\theta_b < 100°$

$$\boxed{C_o = f\left(\frac{L_o}{D}, K_b^*\right)}$$ ([1] figure 9.4)



- $\theta_b \geq 100°$

  $\boxed{C_o = 1}$ (negligible effect)

■ correction factor for rectangular cross-section

- b/W < 0.7 and Lo/D > 1

  $$\boxed{C_{or} = 1 - \frac{1 - C_o}{2}}$$

- b/W > 1 and Lo/D < 1
  - ♦ 1.5 < r/W < 3

    $\boxed{C_{or} = 2}$
  - ♦ r/W ≤ 1.5 or r/W ≥ 3

    $\boxed{C_{or} = C_o}$

- otherwise

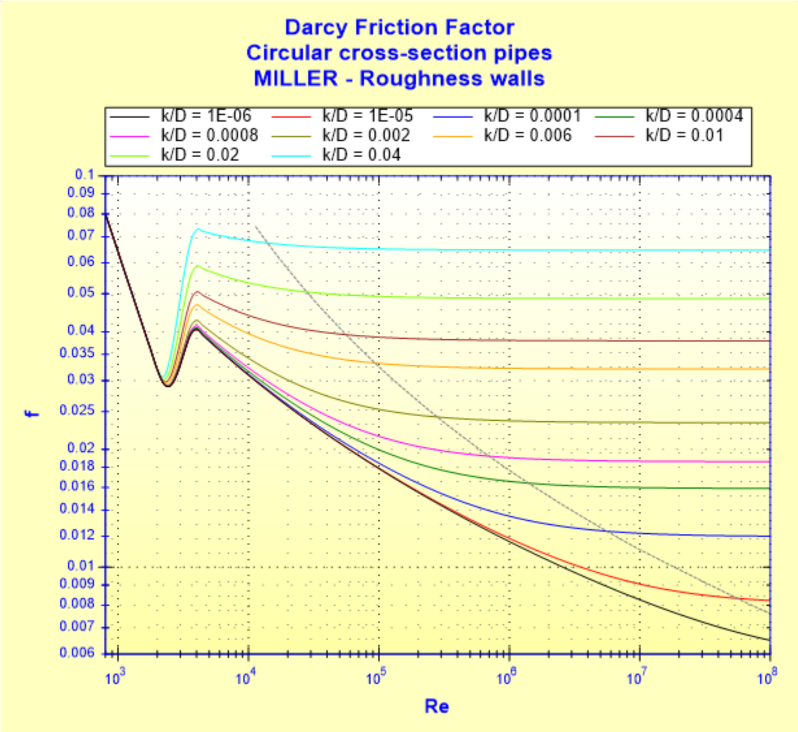
  $\boxed{C_{or} = C_o}$

If this option is not activated, the factors C_o and C_{or} are equal to unity.

Darcy friction factor:

$$\boxed{f = f\left(\mathrm{Re}, \frac{k}{D}\right)}$$

See Straight Pipe - Rectangular Cross-Section and Roughness Walls (MILLER)



Roughness correction factor:

$$\boxed{C_f = \frac{f_{rough}}{f_{smooth}}} \qquad \text{([1] equation 9.3)}$$

with:

f_{rough}: Darcy friction factor for rough pipe at Re

f_{smooth}: Darcy friction factor for smooth pipe (k = 0) at Re

For Re > 10^6, C_f is calculated from equation (9.3) for Re = 10^6

Corrected loss coefficient for the first bend:

$$\boxed{K_{b1} = K_b^* \cdot C_{Re} \cdot C_f} \qquad \text{([1] equation 9.4)}$$

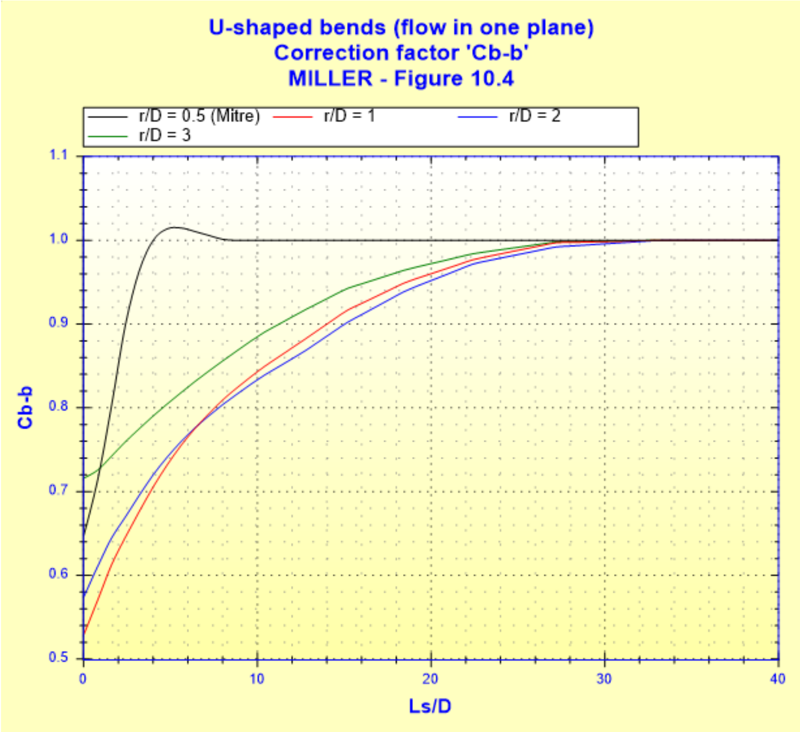
Corrected loss coefficient for the second bend:

$$\boxed{K_{b2} = K_b^* \cdot C_{Re} \cdot C_f \cdot C_o} \qquad \text{([1] equation 9.4)}$$

Interaction correction factor:

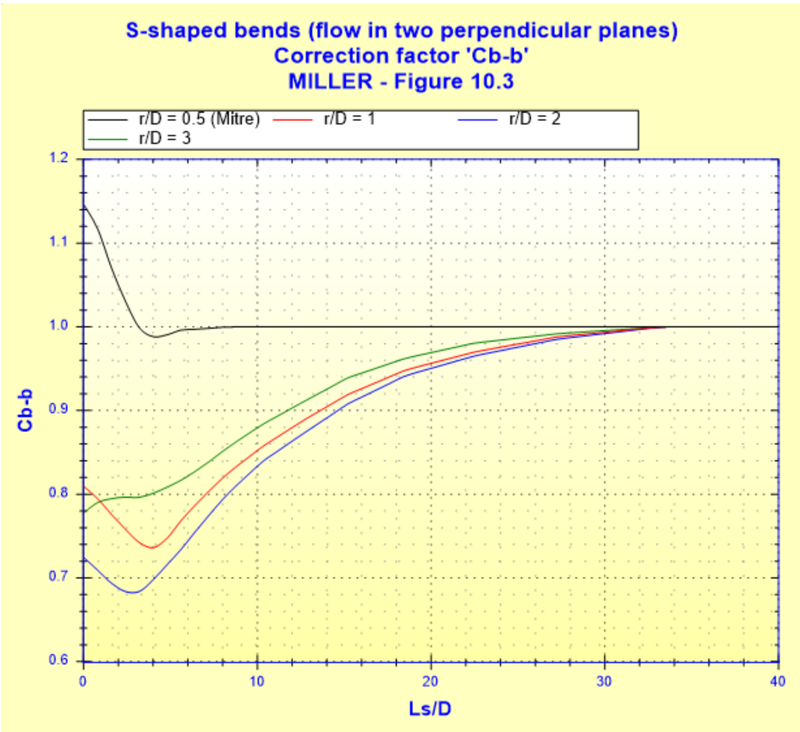
$$\boxed{C_{b-c} = f\left(\frac{L_s}{D}, \frac{r}{D}\right)}$$

- Aspect ratio b/W < 0.7



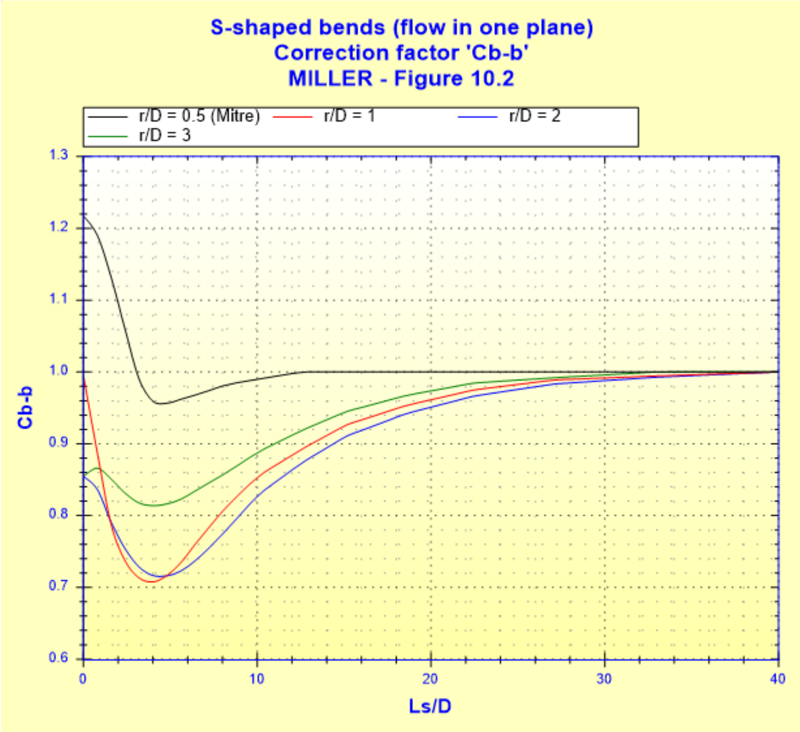
([1] figure 10.4)

■ Aspect ratio 0.7 ≤ b/W ≤ 1.5



([1] figure 10.3)

■ Aspect ratio b/W > 1.5



([1] figure 10.2)

Pressure loss coefficient of the two bends:

- Aspect ratio b/W < 0.7

$$K_{b-b} = (K_{b1} + K_{b2}) \cdot \left[1 - \frac{1 - C_{b-b}}{2}\right]$$

([1] equation 10.1 with correction factor § 10.4)

- Aspect ratio b/W ≥ 0.7

$$K_{b-b} = (K_{b1} + K_{b2}) \cdot C_{b-b}$$

([1] equation 10.1)

Friction loss coefficient of the straight length between bends:

$$K_f = f_{rough} \cdot \frac{L_s}{D}$$

([1] equation 8.3)

Total pressure loss coefficient (based on the mean velocity in the bend):

$$K = K_{b-b} + K_f$$

Total pressure loss (Pa):

$$\Delta P = K \cdot \frac{\rho \cdot U^2}{2}$$

([1] equation 8.1b)

Total head loss of fluid (m):

$$\Delta H = K \cdot \frac{U^2}{2 \cdot g}$$

([1] equation 8.1a)

Hydraulic power loss (W):

$$Wh = \Delta P \cdot Q$$

Straight length of equivalent pressure loss (m):

$$\boxed{L_{eq} = K \cdot \frac{D}{f_{rough}}}$$

Symbols, Definitions, SI Units:

W	Cross-section height (m)
b	Cross-section width (m)
D	Bend hydraulic diameter (m)
A	Cross-section area (m^2)
Q	Volume flow rate (m^3/s)
G	Mass flow rate (kg/s)
U	Mean velocity (m/s)
L_s	Straight length between the two bends (m)
L	Total length measured along the axis (m)
r	Radius of curvature (m)
θ_b	Curvature angle of each bend (°)
V	Fluid volume (m^3)
M	Fluid mass (kg)
Re	Reynolds number ()
K_b^*	Basic loss coefficient ()
C_{Re}	Reynolds number correction factor ()
C_o	Outlet pipe length correction factor for circular cross-section ()
C_{or}	Outlet pipe length correction factor for rectangular cross-section ()
L_0	Length of the straight section downstream of the bend (m)
f	Darcy friction factor ()
k	Absolute roughness of walls (m)
C_f	Roughness correction factor ()
K_{b1}	Corrected loss coefficient for the first bend ()
K_{b2}	Corrected loss coefficient for the second bend ()
$C_{b\text{-}b}$	Interaction correction factor ()
$K_{b\text{-}b}$	Pressure loss coefficient of the two bends ()
K_f	Friction loss coefficient of the straight length between bends ()
K	Total pressure loss coefficient (based on the mean velocity in the bend) ()
ΔP	Total pressure loss (Pa)
ΔH	Total head loss of fluid (m)
Wh	Hydraulic power loss (W)
L_{eq}	Straight length of equivalent pressure loss (m)
ρ	Fluid density (kg/m^3)
ν	Fluid kinematic viscosity (m^2/s)
g	Gravitational acceleration (m/s^2)

Validity range:

- turbulent flow regime ($Re \geq 10^4$)

- stabilized flow upstream bend
- radius of curvature / hydraulic diameter ratio 'r/D': greater than or egal to 0.5 and lower than or egal to 10

  for ratio 'r/D' greater than 3, the pressure loss coefficient 'K' is estimated by taking into account the interaction correction factor 'Cb-b' corresponding to a ratio 'r/D' of value 3.

- curvature angle: between 10° and 180°

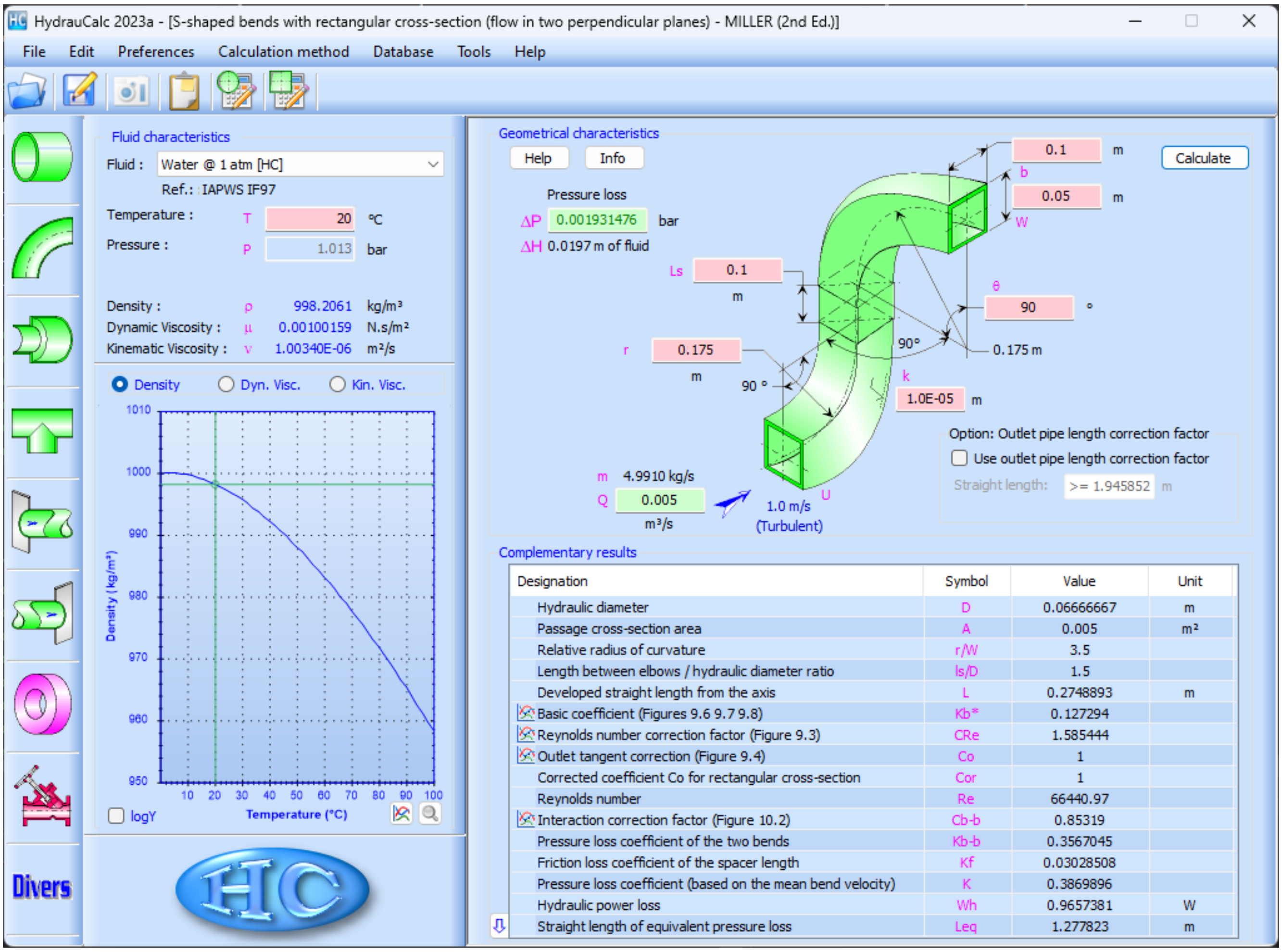
  the interaction correction factor 'Cb-b' is applicable for angles 'θ_b' between 70° and 90°.

  for angles 'θ_b' less than 70° or greater than 90°, the pressure loss coefficient 'K' is estimated by taking into account the interaction factor coefficient 'Cb-b' applicable to angles 'θ_b' between 70° and 90°

- relative radius of curvature 'r/W': between 0.5 and 10
- sides ratio 'b/W': between 0.5 and 2

  note: for any sides ratio 'b/W' less than 0.5, the resistance coefficients K_b^* are obtained by linear extrapolation from the values of K_b^* calculated for sides ratios of 0.5 and 1.
  for any sides ratio 'b/W' greater than 0.5 and 2, the resistance coefficients K_b^* are obtained by linear extrapolation from the values of K_b^* calculated for sides ratios of 1 and 2.

Example of application:



References:

[1] Internal Flow System, Second Edition, D.S. Miller