# Gradual Expansion Circular Cross-Section (MILLER) 



## Model description:

This model of component calculates the head loss (pressure drop) generated by the flow in a gradual expansion.

The head loss by friction in the inlet and outlet piping is not taken into account in this component.

## Model formulation:

Half top angle of cone $\left({ }^{\circ}\right)$ :
$\theta=\tan ^{-1}\left(\frac{D_{2}-D_{1}}{2 \cdot N}\right)$

Minor cross-sectional area ( $m^{2}$ ):
$\mathrm{A}_{1}=\pi \cdot \frac{D_{1}^{2}}{4}$

Major cross-sectional area $\left(m^{2}\right)$ :
$\mathrm{A}_{2}=\pi \cdot \frac{D_{2}{ }^{2}}{4}$

Mean velocity in minor diameter ( $\mathrm{m} / \mathrm{s}$ ):

$$
U_{1}=\frac{Q}{A_{1}}
$$

Mean velocity in major diameter ( $\mathrm{m} / \mathrm{s}$ ):

$$
U_{2}=\frac{Q}{A_{2}}
$$

Fluid volume in the truncated cone $\left(m^{3}\right)$ :

$$
\mathrm{V}=N \cdot \frac{\pi}{3} \cdot\left(\left(\frac{D_{1}}{2}\right)^{2}+\left(\frac{D_{2}}{2}\right)^{2}+\left(\frac{D_{1}}{2}\right) \cdot\left(\frac{D_{2}}{2}\right)\right)
$$

Fluid mass in the truncated cone (kg):

$$
\mathrm{M}=V \cdot \rho
$$

Reynolds number in minor diameter:

$$
\operatorname{Re}_{1}=\frac{U_{1} \cdot D_{1}}{v}
$$

Reynolds number in major diameter:

$$
\operatorname{Re}_{2}=\frac{U_{2} \cdot D_{2}}{v}
$$

## Local resistance coefficient:

## - $\operatorname{Re}_{1} \geq 10^{4}$

$K_{* d}=f\left(\frac{N}{D_{1} / 2}, \frac{A_{2}}{A_{1}}\right)$

## ([1] figure 11.5+)

Gradual expansion Coefficient of local resistance ( $\mathrm{K}^{*} \mathrm{~d}$ ) MILLER - Figure 11.5+ ( $\mathrm{Re}>=1 \mathrm{e} 6$ )


- $\operatorname{Re}_{1}<10^{4}$

$$
K_{\text {lam }}=f\left(K_{\text {turb }}, \operatorname{Re}_{1}\right)
$$

([1] figure 14.31)
where:
$K_{\text {turb }}$ is the local resistance coefficient in turbulent regime ( $K{ }_{\star d}$ for $R e_{1}=10^{4}-$ figure 11.5+)

Laminar loss coefficient relationship to turbulent loss coefficient
MILLER - Figure $14.31(\mathrm{Re}<1 \mathrm{e} 4)$


Reynolds Number Correction ( $\mathrm{Re}_{1}<10^{4}$ ):

$$
C_{\mathrm{Re}}=\frac{K_{\text {lam }}}{K_{\text {turb }}}
$$

Total pressure loss coefficient (based on mean velocity in minor diameter):

- turbulent flow $\left(\operatorname{Re}_{1} \geq 10^{4}\right)$ :

$$
K=K_{*} d
$$

- laminar flow $\left(\mathrm{Re}_{1}<10^{4}\right)$ :

$$
K=K_{l a m}
$$

Total pressure loss (Pa):

$$
\Delta P=K \cdot \frac{\rho \cdot U_{1}^{2}}{2}
$$

Total head loss of fluid (m):

$$
\Delta H=K \cdot \frac{U_{1}^{2}}{2 \cdot g}
$$

Hydraulic power loss (W):
$W h=\Delta P \cdot Q$

## Symbols, Definitions, SI Units:

$D_{1} \quad$ Minor diameter (m)

| $\mathrm{D}_{2}$ | Major diameter (m) |
| :---: | :---: |
| $N$ | Truncated cone length (m) |
| $\theta$ | Half top angle of cone ( ${ }^{\circ}$ ) |
| $A_{1}$ | Minor cross-sectional area ( $\mathrm{m}^{2}$ ) |
| $A_{2}$ | Major cross-sectional area ( $\mathrm{m}^{2}$ ) |
| Q | Volume flow rate ( $\mathrm{m}^{3} / \mathrm{s}$ ) |
| $U_{1}$ | Mean velocity in minor diameter ( $\mathrm{m} / \mathrm{s}$ ) |
| $U_{2}$ | Mean velocity in major diameter ( $\mathrm{m} / \mathrm{s}$ ) |
| G | Mass flow rate (kg/s) |
| V | Fluid volume in the truncated cone ( $\mathrm{m}^{3}$ ) |
| M | Fluid mass in the truncated cone (kg) |
| $\mathrm{Re}_{1}$ | Reynolds number in minor diameter () |
| $\mathrm{Re}_{2}$ | Reynolds number in major diameter () |
| $K^{*}{ }_{\text {d }}$ | Local resistance coefficient for $\mathrm{Re}_{1} \geq 10^{4}$ () |
| K turb | Local resistance coefficient for $\mathrm{Re}_{1}=10^{4}$ () |
| Klam | Local resistance coefficient for $\mathrm{Re}_{1}<10^{4}$ () |
| $C_{\text {Re }}$ | Reynolds number correction () |
| K | Total pressure loss coefficient (based on mean velocity in minor diameter) () |
| $\Delta P$ | Total pressure loss (Pa) |
| $\Delta \mathrm{H}$ | Total head loss of fluid (m) |
| Wh | Hydraulic power loss (W) |
| $\rho$ | Fluid density ( $\mathrm{kg} / \mathrm{m}^{3}$ ) |
| $v$ | Fluid kinematic viscosity ( $\mathrm{m}^{2} / \mathrm{s}$ ) |
| 9 | Gravitational acceleration ( $\mathrm{m} / \mathrm{s}^{2}$ ) |

## Validity range:

- any flow regime: laminar and turbulent
- area ratio (A2/A1) between 1.1 and 4
- contraction length ratio ( $N /(D 1 / 2$ )) less than 20
note: for Reynolds number " $\mathrm{Re}_{1}$ " lower than $10^{4}$, and coefficients "Kturb" lower than 0.5 or greater than 10, the laminar pressure loss coefficient "Kam" is extrapolated

Example of application:


## References:

[1] Internal Flow System, Second Edition, D.S. Miller

HydrauCalc
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