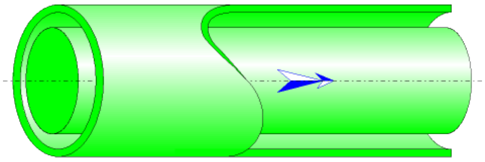
Straight Pipe
Annular Cross-Section and Roughness Walls
(MILLER)



Model description:

This model of component calculates the major head loss (pressure drop) of a horizontal straight pipe of annular and constant cross-section.
In addition, the flow is assumed fully developed and stabilized.

The head loss is due to the friction of the fluid on the inner walls of the piping and is calculated with the Darcy formula.

Darcy friction factor is determined:

- for laminar flow regime by the law of Hagen-Poiseuille (independent of the value of relative roughness),
- for turbulent flow regime by the explicit Swamee-Jain equation (dependent of the value of relative roughness), the explicit Swamee-Jain equation is a Colebrook-White equation approximation,
- for critical flow regime by interpolation between friction factors of laminar and turbulent flow.

Model formulation:

Hydraulic diameter (m):

$$\mathrm{D} = d_0 - d_1$$

Cross-section area (m²):

$$\mathrm{A} = \pi \cdot \frac{d_0^{\,2} - d_1^{\,2}}{4}$$

Mean velocity (m/s):

$$U = \frac{Q}{A}$$

Mass flow rate (kg/s):

$$m = Q \cdot \rho$$

Fluid volume in the pipe (m^3):

$$V = A \cdot L$$

Fluid mass in the pipe (kg):

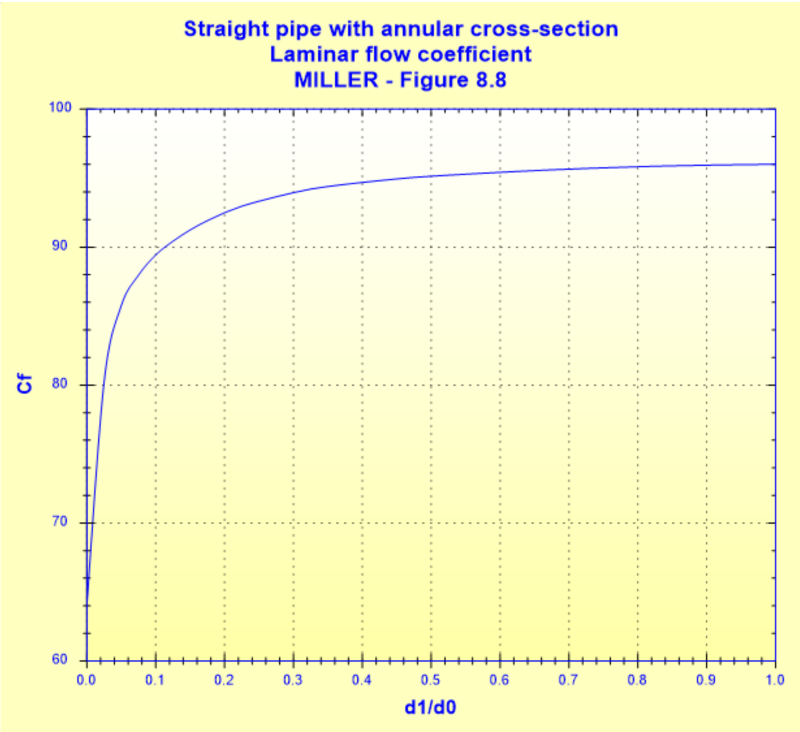
$$M = V \cdot \rho$$

Reynolds number:

$$\mathrm{Re} = \frac{U \cdot D}{\nu}$$

Laminar flow coefficient:

$$Cf = f(d_1 / d_0)$$ ([1] figure 8.8)

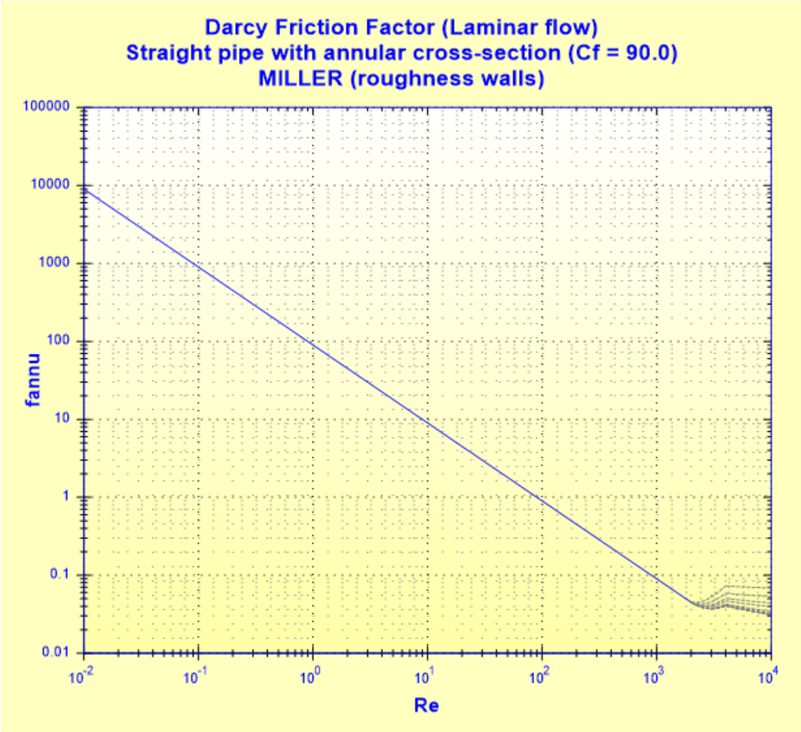


Darcy friction factor:

- laminar flow regime (Re ≤ 2000):

Darcy friction factor for annular cross-section:

$$f_{annu} = \frac{Cf}{\mathrm{Re}}$$ ([1] equation 8.7)



([1] equation 8.7with Cf = 90)

■ turbulent flow regime – transition region and complete turbulence region (Re ≥ 4000):

Darcy friction factor for circular cross-section:

Swamee-Jain equation (Colebrook-White equation approximation)

$$f_{circ} = \frac{0.25}{\left[\log\left(\frac{k}{3.7 \cdot D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2}$$

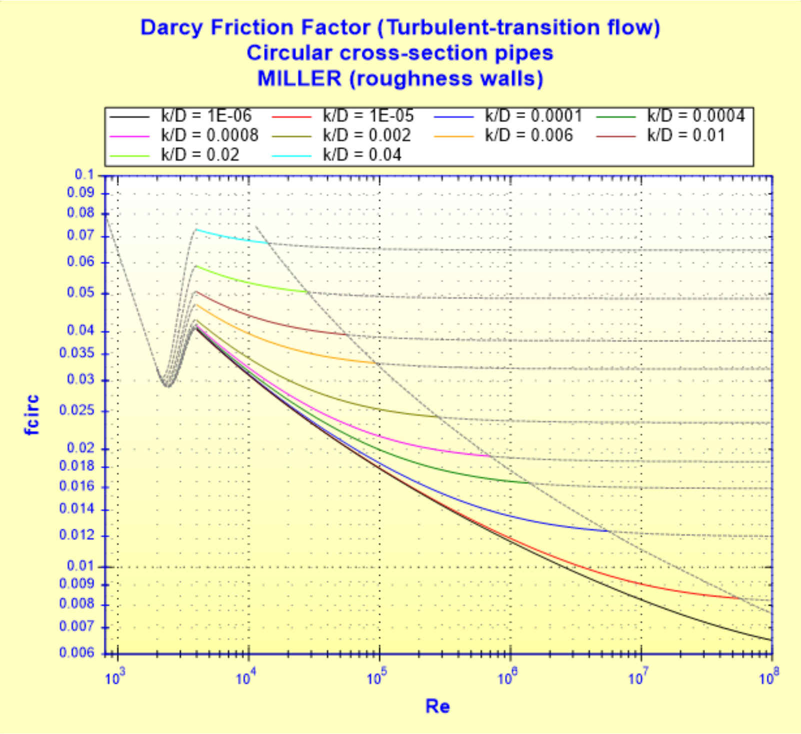
([1] equation 8.4)

Reynolds number corresponding to the beginning of complete turbulence:

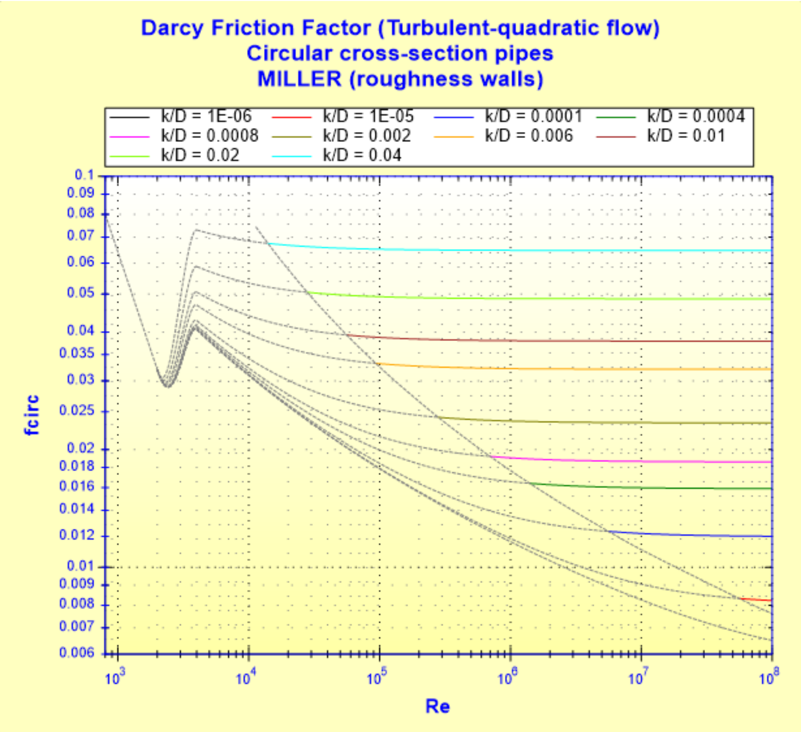
$$\mathrm{Re}''_{lim} = \frac{560}{k/D}$$

([2] diagram 2.4)

Transition region



Complete turbulence region



Darcy friction factor for annular cross-section:

$$f_{annu} = 1.05 \cdot f_{circ}$$ ([1] equation 8.5)

- critical flow regime (2000 < Re < 4000):

Darcy friction factor for circular cross-section:

cubic interpolation

$$f_{circ} = (X1 + R \cdot (X2 + R \cdot (X3 + X4)))$$ ([3])

with:

$$R = \frac{Re}{2000}$$

$$X1 = 7 \cdot FA - FB$$

$$X2 = 0.128 - 17 \cdot FA + 2.5 \cdot FB$$

$$X3 = -0.128 + 13 \cdot FA - 2 \cdot FB$$

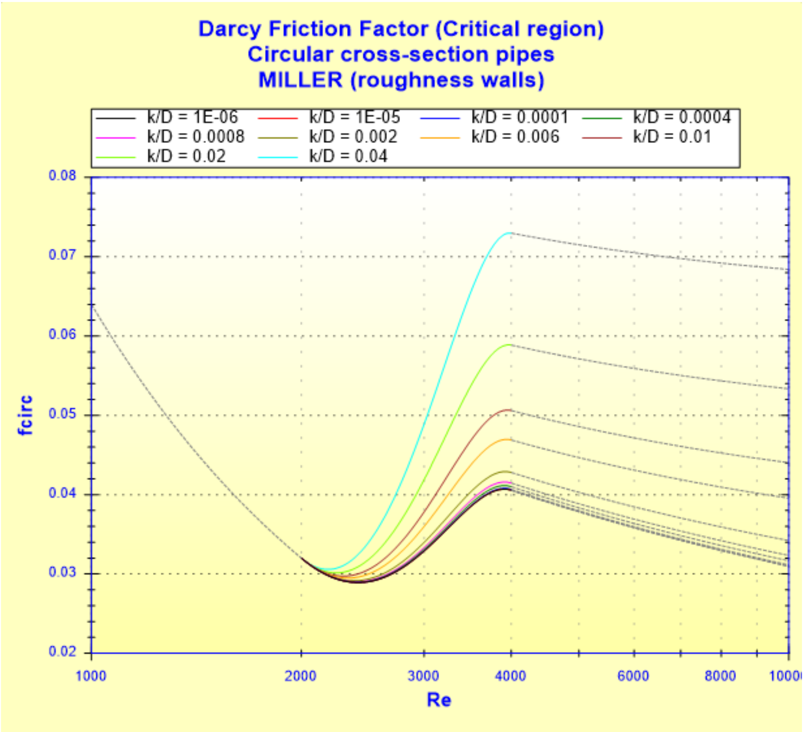
$$X4 = R \cdot (0.032 - 3 \cdot FA + 0.5 \cdot FB)$$

$$FA = Y3^{-2}$$

$$FB = FA \cdot \left(2 - \frac{0.00514215}{Y2 \cdot Y3}\right)$$

$$Y2 = \frac{k}{3.7 \cdot D} + \frac{5.74}{Re^{0.9}}$$

$$Y3 = -0.86859 \cdot ln\left(\frac{k}{3.7 \cdot D} + \frac{5.74}{4000^{0.9}}\right)$$



Darcy friction factor for annular cross-section:

$$f_{annu} = 1.05 \cdot f_{circ}$$ ([1] equation 8.5)

Friction pressure loss coefficient:

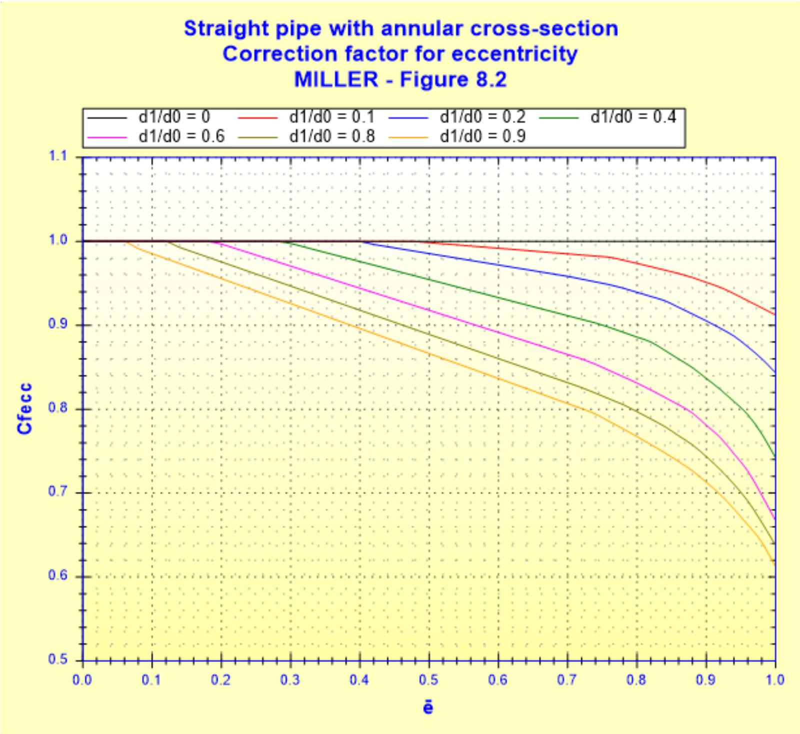
$$K_f = f_{annu} \cdot \frac{L}{D}$$ ([1] equation 8.3)

Relative eccentricity:

$$\bar{e} = \frac{2 \cdot e}{d_1 - d_0}$$

Correction for axis eccentricity:

$$Cf_{ecc} = f\left(\bar{e}, d_1/d_0\right)$$ ([1] figure 8.2)



Total pressure loss coefficient (based on the mean pipe velocity):

$$K = K_f \cdot Cf_{ecc}$$

Total pressure loss (Pa):

$$\Delta P = K \cdot \frac{\rho \cdot U^2}{2}$$

([1] equation 8.1b)

Total head loss of fluid (m):

$$\Delta H = K \cdot \frac{U^2}{2 \cdot g}$$

([1] equation 8.1a)

Hydraulic power loss (W):

$$Wh = \Delta P \cdot Q$$

Symbols, Definitions, SI Units:

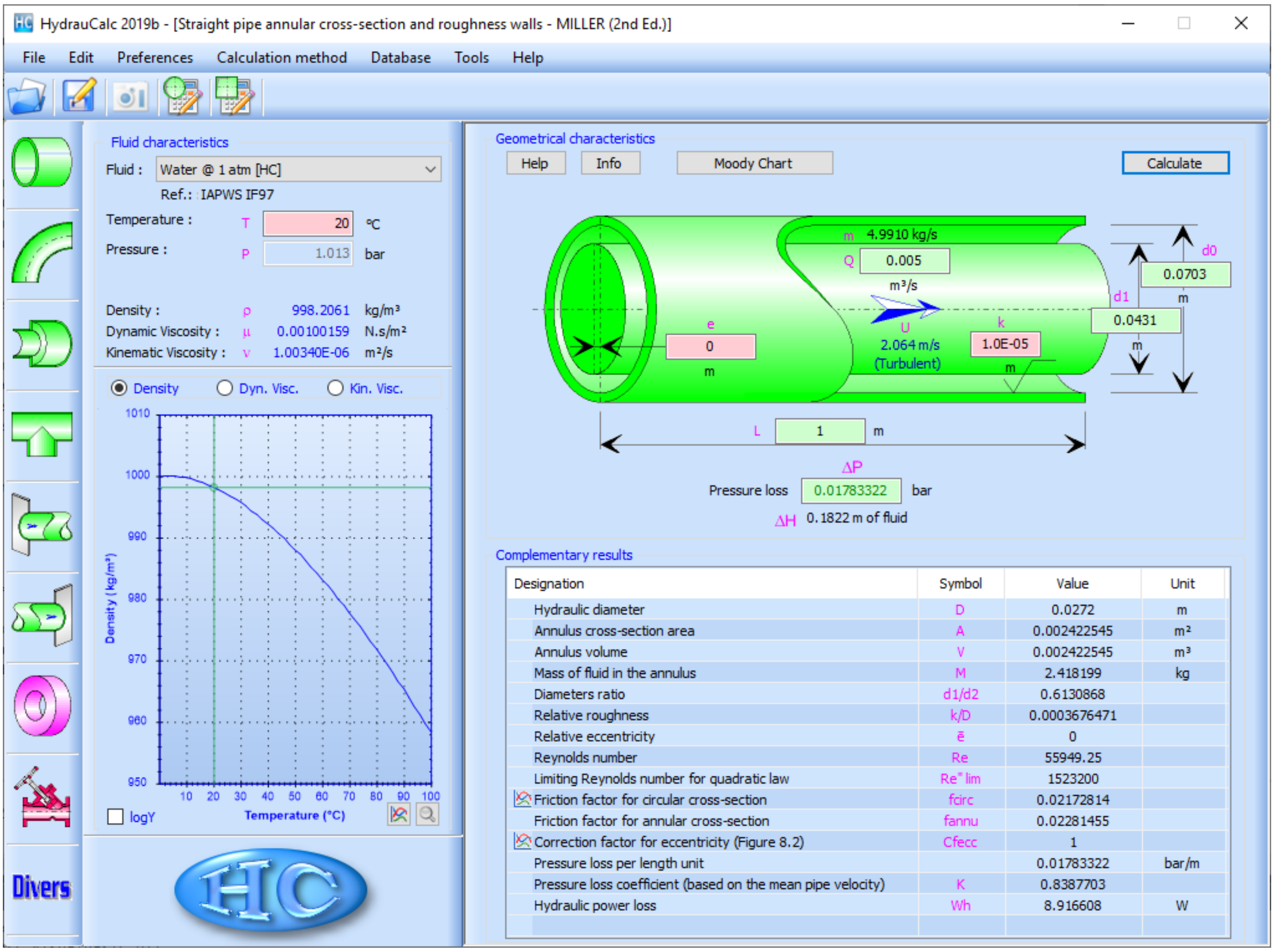
d_1	Annulus inner diameter (m)
d_0	Annulus outer diameter (m)
D	Hydraulic diameter (m)
A	Cross-section area (m^2)
Q	Volume flow rate (m^3/s)
U	Mean velocity (m/s)
m	Mass flow rate (kg/s)
L	Pipe length (m)
V	Fluid volume in the pipe (m^3)
M	Fluid mass in the pipe (kg)
Re	Reynolds number ()
Cf	Laminar flow coefficient ()

k	Absolute roughness of walls (m)
f_{circ}	Darcy friction factor for circular cross-section ()
Re''_{lim}	Reynolds number corresponding to the beginning of complete turbulence ()
f_{annu}	Darcy friction factor for annular cross-section ()
K_f	Friction pressure loss coefficient ()
e	Pipes eccentricity (m)
$\bar{e}$	Relative eccentricity ()
Cf_{ecc}	Correction for axis eccentricity ()
K	Total pressure loss coefficient (based on the mean pipe velocity) ()
ΔP	Total pressure loss (Pa)
ΔH	Total head loss of fluid (m)
Wh	Hydraulic power loss (W)
ρ	Fluid density (kg/m³)
ν	Fluid kinematic viscosity (m²/s)
g	Gravitational acceleration (m/s²)

Validity range:

- any flow regime: laminar, critical and turbulent ($Re \leq 10^8$)
- relative roughness $k/D \leq 0.05$
- stabilized flow
- relative eccentricity ≤ 0.9
  note: for relative eccentricity greater than 0.9, the correction factor for axis eccentricity 'Cfecc' is extrapolated

Example of application:



References:

[1] Internal Flow System, Second Edition, D.S. Miller (1990)

[2] Handbook of Hydraulic Resistance, 3rd Edition, I.E. Idelchik (2008)

[3] Dunlop (1991)